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California Institute of Technology

NAG 2-792

Robust Nonlinear Control
of Vectored Thrust Aircraft

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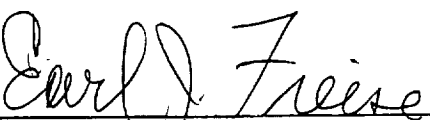
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June 1, 1992 to May 31, 1993


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(NASA-CR-192727) ROBUST NONLINEAR
CONTROL OF VECTORED THRUST AIRCRAFT
(California Inst. of Tech.) 44 p

N93-25199

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Notes for

**Robust Nonlinear Control of
Vectored Thrust Aircraft**

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Abstract

This note outlines an interdisciplinary program in robust control for nonlinear systems with applications to a variety of engineering problems. Major emphasis will be placed on flight control, with both experimental and analytical studies.

This program builds on recent new results in control theory for stability, stabilization, robust stability, robust performance, synthesis, and model reduction in a unified framework using Linear Fractional Transformations (LFTs), Linear Matrix Inequalities (LMIs), and the structured singular value μ . Most of these new advances have been accomplished by the Caltech controls group independently or in collaboration with researchers in other institutions. These recent results offer a new and remarkably unified framework for all aspects of robust control, but what is particularly important for this program is that they also have important implications for system identification and control of nonlinear systems. This combines well with Caltech's expertise in nonlinear control theory, both in geometric methods and methods for systems with constraints and saturations.

1 Summary of proposed research

1.1 Introduction

In this note we outline a comprehensive research program in robust control for nonlinear system. Our research in control theory will build on our recent progress in robust control, robust identification, and nonlinear control. Our long-term goals are:

1. Develop robust control methods for nonlinear systems that integrate the entire controller design process, including modeling and system identification, as well as online adaptation, testing, evaluation, and redesign. Also, integrate the control design process more fully with sensor, actuator, and plant design.
2. Apply developing robust control methods to application areas to motivate the theory and provide context. Experimental projects include flight control, combustion instabilities, compressor surge, and several other control problems in mechanical and chemical process systems. More speculative applications in control of vortex shedding from bluff bodies and configuration and signal processing for microsensors are also being considered.

We are proposing an uncommonly synergistic relationship between control theory and applications, which we believe greatly benefits both. In addition to motivating the theory and providing context for its development, the theory can often be advanced much more rapidly by first considering domain specific rather than general systems.

1.2 Robust control of nonlinear systems

One long term goal of this program is to integrate the entire controller design process, from modeling and identification to analysis, simulation, and synthesis through to implementation, including online adaptation, testing, evaluation and redesign. A more systematic design procedure reduces the design effort, broadens the applicability, improves the performance and, in general, makes the affected industrial sectors more competitive. While recent developments in robust control theory have had a very positive impact on analysis and synthesis of controllers, there remain substantial portions of the total process which are far from systematic. On one end of the design process, the modeling for control design is as much art as science. On the other end, the design of the final controller is often completed with substantial additional logic that is added in a completely *ad hoc* manner.

Nonlinearities, constraints, and uncertainty

The greatest obstacles to practical application of advanced control theory have traditionally been the presence of nonlinearities, operating constraints, and model uncertainty. Until the last decade or so, modern and classical control theory addressed none of these issues in a satisfactory manner. Since then, great research progress has been made in addressing these issues *separately* in a fundamental manner, but no method systematically addresses more than one. For example, robust control has focused on the issue of model uncertainty, with tremendous success by almost any measure. So far, the most successful applications of robust control techniques such as μ analysis and synthesis have occurred in problem domains (flexible structures, flight control, distillation) where there may be substantial uncertainty in the available models, but the basic structure of the system is understood, the uncertainty can be quantified, and the nonlinearities can be bounded and treated as

perturbations on a nominal linear model. The degrees of freedom and the dimension of the input, output, and state may be extremely high. In such cases, direct intuition about control strategies is almost useless, and the power of sophisticated synthesis methods based on advanced mathematics is dramatically demonstrated.

Similarly, Model Predictive Control (MPC) has focused on control design for otherwise linear systems with constraints and has found substantial applications, particularly in process control. However, some operational requirements are impossible to express in the MPC fashion, i.e., through a single objective function which is to be optimized on-line subject to a set of constraints. Also, the scope of the available techniques for robust stability analysis is extremely limited. There is no accepted definition of robust performance, even less a corresponding synthesis technique. Furthermore, even linear MPC with linear constraints is computationally too complex to be used for demanding high-speed applications like compressor surge control or aircraft turbine control. The idea extends to nonlinear systems in principle, but we understand very little about its properties and the computational problems are enormous.

Advances in nonlinear control theory have resulted in a much more detailed understanding of the geometry of nonlinear control systems and the interaction between geometric properties and control design. The primary emphasis in nonlinear control design has focused on mathematical analysis and not engineering applications. Recent techniques in feedback linearization and dynamic inversion have found use in some practical applications, but the limited class of systems to which these methods apply leaves much room for research. There are currently no nonlinear design tools which simultaneously take into account performance specifications, disturbance rejection, and model uncertainty.

Unfortunately, nonlinearities, operating constraints, and model uncertainty often occur simultaneously, and thus many practical control applications fail to take advantage of theoretical advances because they are dominated by combinations of these issues. Engineers address these issues with domain-specific approaches that have evolved through trial and error, and simple schemes that rely heavily on engineering intuition. It is thus not surprising that traditional schemes still dominate the application world. For a PID controller, we know how to introduce anti-windup to preserve stability and minimize performance deterioration in the event of manipulated variable saturation and when controller switching is dictated by a selector. We also know quite well how to tune PID controllers to be insensitive to process nonlinearities and model uncertainty, and we know when simple nonlinear compensators are needed. Unfortunately, these simple, practical schemes for PID controllers can be overwhelmed by complex, multivariable systems

There is an established industrial need for multivariable controllers which can deal with nonlinearities, constraints and uncertainty, which can be switched in and out as demanded by logic elements, and which can be easily gain-scheduled, simulated, verified, and implemented. The new multivariable controller design techniques will not fulfill their promise unless the important issues of process nonlinearity, constraint handling and model uncertainty are resolved. Our proposed program will establish a sound theoretical basis for control design methodologies addressing all three issues.

Modeling and system identification for control design

Another aspect of control design that remains far too unsystematic and which we are focusing on is modeling and identification of uncertain dynamic systems from first principle models and experimental data for the purpose of robust control. Recent advances in robust control allow control system designers to replace ad hoc trial and error approaches for dealing with uncertainty

with explicit and systematic analytical methods, allowing for the use of much more realistic models and performance objectives. The enormous flexibility of these methods makes them very powerful, but at the same time puts a new burden on the engineer to model and identify uncertainties, both from first principles and experimental data.

The technology to produce uncertainty descriptions of dynamical systems has not kept pace with the techniques to analyze the resulting models, and thus modeling and identification for control design continues to involve substantial ad hoc aspects. New robust control design techniques actually require more sophisticated software support than ever before. When designers had few choices there were few decisions to make. With more powerful analytical tools to deal with more complex models, uncertainty descriptions, and multiple competing performance objectives, the designer can be overwhelmed by the choices. We will pursue two approaches to narrowing these choices: 1) more direct connection with modeling and data and 2) more sophisticated software to manage the complex decision making.

Beyond robust control

The control theory portion of our proposed research program builds on some exciting new results in stability, stabilization, robust stability, robust performance, synthesis, and model reduction in a unified framework using Linear Fractional Transformations (LFTs), Linear Matrix Inequalities (LMI), and the structured singular value μ . At the heart of our approach to robust control is the observation that multidimensional and uncertain systems are naturally written as LFTs, and essentially all of the standard questions in robust control are easily and naturally expressed using LFTs. The proposed research program will pursue this LFT/LMI approach to robust control as well as establish additional connections with nonlinear control theory.

Perhaps as important as the specific results themselves is the technical framework in which they are developed. It offers a new and remarkably unified framework for all aspects of robust control, including analysis, synthesis, and model reduction. What is particularly important for this program is that it also promises to have important implications for system identification and control of nonlinear systems. Remarkably, it also greatly streamlines the development of even the standard results, threatening to make the most recent conventional treatments of balanced truncation, stabilization, and state-space H_∞ obsolete.

While these results are only beginning to be applied to any serious engineering problems, their potential impact is enormous. For example, the model reduction results are the first techniques for producing reduced order models of uncertain systems with guaranteed error bounds. The generalizations of the stabilization and H_∞ theory should provide reliable, systematic methods for gain-scheduling robust controllers. By the beginning of this program we will have preliminary application experience to provide some engineering perspective on the new results. An important aspect of this proposed program is the evaluation of these techniques on several experimental systems that are describe in greater detail later.

Adaptive control

From the point of view of the proposed research program, adaptive control is a scheme for designing nonlinear controllers for uncertain systems. Essentially all successful practical adaptive controllers have been designed by combining good system identification with good control design. Adaptive control will naturally evolve in our research program from advances in robust system identification and robust controller synthesis. We believe that this is the best possible research direction to develop systematic methods for the design of practical adaptive control schemes.

Integrated controller/plant design

While we expect this research program to greatly advance design of robust controllers for nonlinear systems, we see control design in a broader context. Increasingly, the role of the control engineer is as much to determine sensor and actuator locations and evaluate the total system performance as it is to design the control system per se. Indeed, in many applications the control engineer's most important function is to help design the basic plant itself, not the actual control design. This is one reason our proposed research program focuses exclusively on analytic methods, since both heuristic-based (e.g. fuzzy logic) and black-box learning (e.g. neural net) methods for controller design offer almost nothing in this setting. While these methods may have some role to play in certain application areas, analytic techniques are always superior in situations where control design must interact substantial with plant design.

1.3 Flight control and related applications

We are also developing several application areas to motivate and provide context for the theory. The largest of these applications will consist of a sequence of experiments in flight control for rotary and fixed wing aircraft. Other applications which are currently being developed, and which will integrate with the research in this proposal, include control of combustion instabilities, control of surge in high performance turbomachinery, control of flexible, articulated structures, and control of vortex shedding in fluid flow processes, and control of various other mechanical and chemical process systems. In each case, the Caltech control group will collaborate with specialists in the application domains.

We plan to establish a directed program of research in design of robust flight control systems, with application in the area of supermaneuverability of high performance jet aircraft. This phase of the research is centered around case studies of the control of several increasingly complex flight control systems: a tethered model helicopter, a vectored thrust engine, and a free-flying, fixed wing aircraft, to be developed jointly with Rockwell, Inc. For this class of systems, the dynamics vary substantially over the operating regions of interest, making use of nonlinear control techniques an attractive possibility. A particular area of interest for flight control systems is in trajectory tracking, especially for trajectories which do not lie in a single operating region and may not remain near an equilibrium point of the system.

Since these systems are extremely difficult to model exactly, disturbances and model mismatch must be taken into account during the design process. Also, it is unlikely that a detailed description of the complete system dynamics will be available. Hence numerical identification of some system characteristics must be performed. Such identification procedures only give approximate descriptions of the plant, and the effect of these approximations must be taken into account in order to guarantee robust performance.

Other application areas in this program with experimental components included surge in high performance turbomachinery and control of flexible articulated structures. More speculative applications in control of vortex shedding from bluff bodies and configuration and signal processing for microsensors will also be considered.

2 Robust Control of Nonlinear Systems

2.1 Outline of proposed research plan

The basic outline of the proposed research program is diagrammed in Figures 1 and 2, divided somewhat artificially into analysis and synthesis. In each case, the top row consists of results which exist or are anticipated to be reasonably well-developed by the start of this program. The flow diagrams then chart the expected evolution of the research program, with time roughly represented by movement down the page. Of course, the plan gets increasingly speculative toward the bottom of the diagram. For example, we have, somewhat tongue-in-cheek, highlighted Nonlinear Robust Adaptive Control at the bottom of the synthesis diagram. We cannot honestly say at this time that we have more than a vague idea about how a truly robust adaptive control theory would look, but we are very confident that our proposed research program has the best chance among currently available theoretical frameworks of getting there.

In many of the boxes indicating research topics there are section numbers listed where a sketch of the key ideas can be found. The arrows indicate roughly the dependence of research topics on results from other research topics. The dotted boxes in the synthesis diagram are boxes from the analysis diagram that are key to progress on the synthesis boxes to which they point.

The remainder of this section will review recent theoretical progress and outline proposed research directions. Additional technical details on the recent results is available in the the references cited. We will begin with a discussion of some generic research issues that effect each topic and then move on to sketches of the individual topics and their relationship to each other.

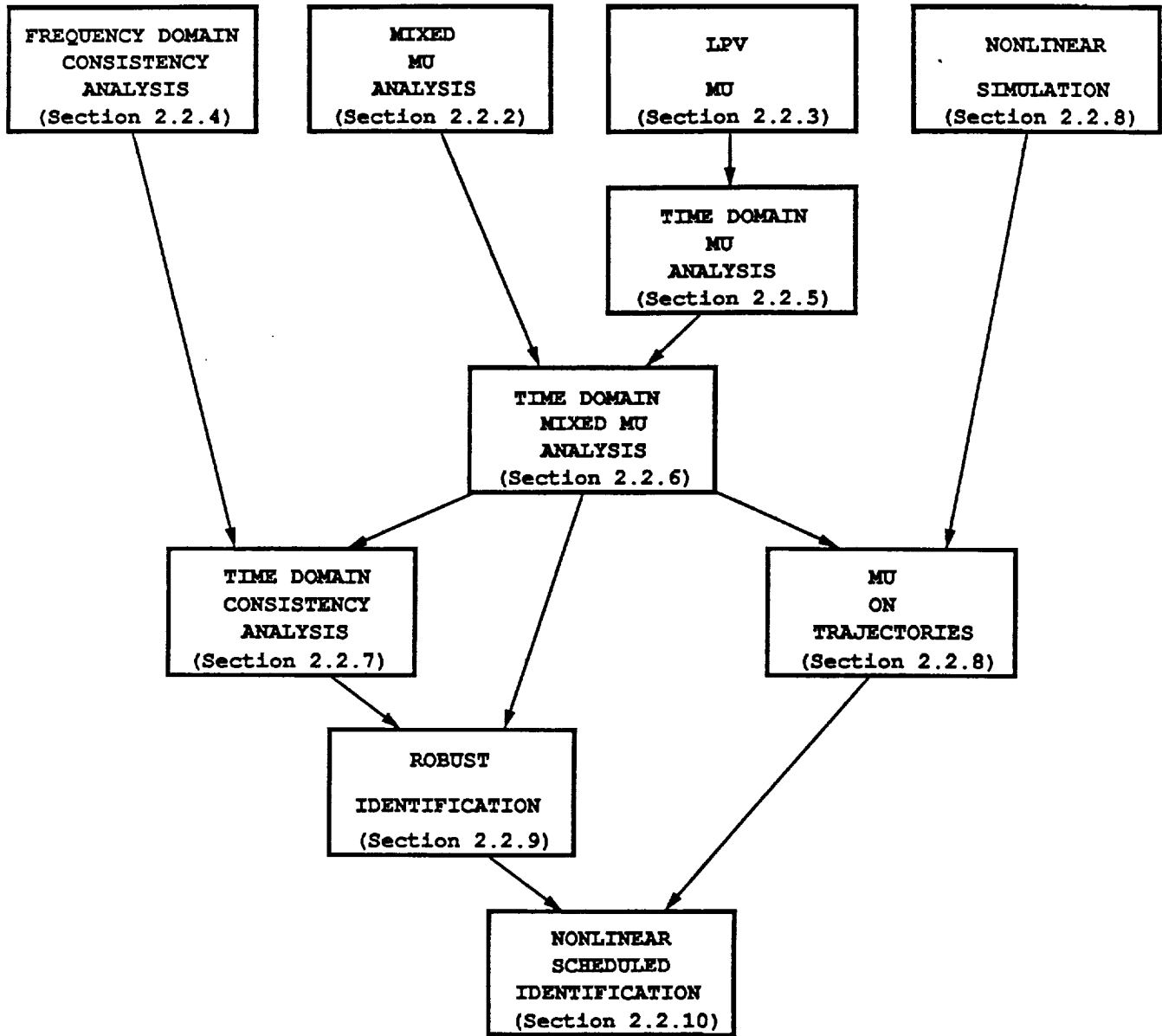
2.1.1 Modeling and uncertainty

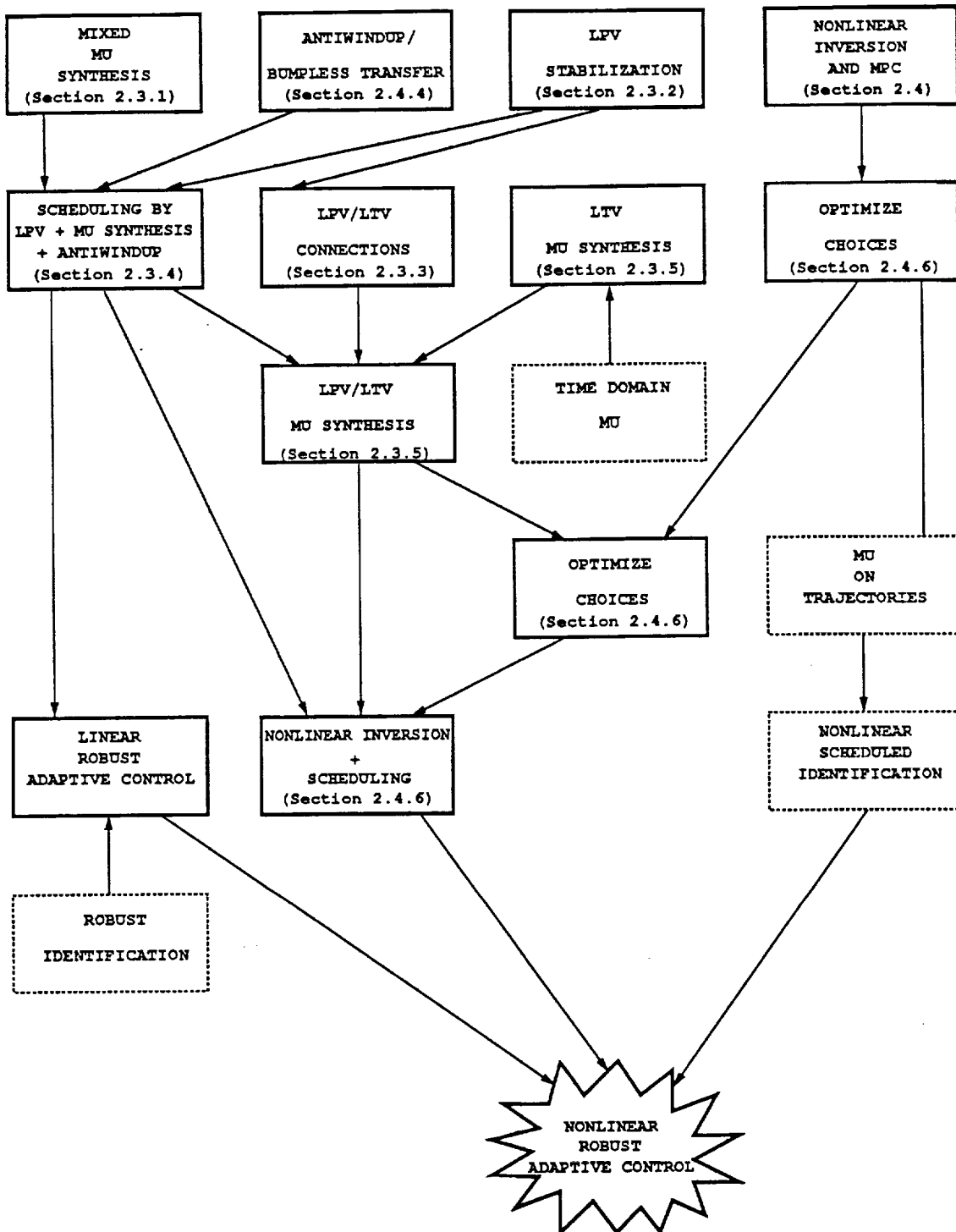
In the field of robust control, there are currently several methodologies which enable one to synthesize controllers which maintain their stability/performance in the presence of perturbations. The number of papers at recent conferences on H_∞ and L_1 optimal control as well as the analysis of real parametric uncertainty attests to the widespread interest in this area. Unfortunately there are relatively few experimental or industrial applications of such techniques. For example, while μ -analysis methods are now used routinely throughout the aerospace industry, H_∞ , L_1 , or μ synthesis methods are just beginning to find substantial application.

While there are many reasons for this lack of applications, an important one is the absence of systematic methods for modeling and identification that are compatible with robust control. Thus, while engineers may be comfortable doing what-if analysis with various uncertainty descriptions, they have no systematic way for obtaining the weighting matrices and uncertainty descriptions required for the synthesis techniques. In contrast, analysis requires much less systematic modeling effort. Engineers can try a variety of uncertainty models and convince themselves that their designs have reasonable robustness properties without committing themselves to any specific uncertainty model.

Most current identification methods attribute uncertainty in the system to additive noise. Robust control design methods require that one account for both unknown (but bounded) dynamic perturbations as well as additive noise. Almost all experimental applications of robust control techniques use ad-hoc methods for identifying a model and its associated uncertainty. Obtaining a description of the uncertainty is an essential part of modeling an uncertain system, and this will be a major emphasis of this research program.

It is now widely recognized that much better connections between modeling, data, and control design and implementation are needed, as evidenced by the excitement generated at the Workshop





on the Modeling of Uncertainty in Control Systems, sponsored by NSF and AFOSR, held in June 1992. While there are certainly intrinsic limitations on the extent to which such connections can be formalized, it is very likely that substantial progress can be made. Several researchers are beginning to look at this broad class of problems, but there is very little in the way of general results. Equally important, there does not exist a reasonable paradigm in which to pose the relevant questions comparable to what has evolved in the areas of robust control or conventional system identification. A major goal of this research program is to follow up on the June workshop in developing a general framework for connecting modeling from first principles, system identification, performance specification, and control design.

2.1.2 Robust Stability and Performance of LFT Systems

There are several notions of robust stability and performance that can be systematically treated in an LFT framework [24]. The most developed now are those involving either L_2 or L_∞ as the underlying signal space for disturbances and performance specifications, as well as for inducing the norms which bound perturbations. The use of L_2 spaces leads to H_∞ norms on systems, and L_∞ leads to what has become known as the L_1 theory. It is also possible to consider perturbations that are from Linear and Time Invariant (LTI) to Nonlinear Time Varying.

Each combination of underlying norm and perturbation model yields a different theory for the resulting robust stability and performance problems. Recently, we have begun to understand in greater detail the relationship between these different theories and what their implications are for control design. The two theories which will be most focused on in this research program both involve L_2 signal spaces. The theory for LTI perturbations will be referred to as the μ theory and the theory for Linear Time Varying (LTV) perturbations (as well as nonlinear) will be referred to as \mathcal{Q} theory.

Current research directions involve deepening our understanding of the relationship between the various uncertainty descriptions and the connections with modeling and data. Our emphasis in this program will be to evaluate recent theoretical work that provides less conservative and more flexible assortments of mathematical descriptions of uncertainty models than was previously available and push the development of new uncertainty models. For example, we plan to develop techniques for dealing with time-varying and/or nonlinear perturbations with bounds on their deviation from LTI as well as bounds on their norm.

2.1.3 Model Reduction of LFT Systems

Model reduction is an issue throughout our proposed research program, as we constantly seek to find simpler descriptions of our problem. Based on the concept of the balanced truncation model reduction method for the one-dimensional linear system, we have developed balanced truncation model reduction method for more general LFTs [75]. An H_∞ norm error bound for the LFT model reduction method has been derived which is a direct generalization of the bounds for single-variable systems. (The new proof is also the simplest yet for the previously known results.) This gives a method for producing reduced order models of both multidimensional systems and uncertain systems with guaranteed error bounds. The notion of balanced grammians is generalized using LMIs.

Current research directions involve further extending model reduction for LFT systems, and in particular, the problems of uncertainty aggregation and component model reduction. We believe both of these problems are extremely difficult and may not yield the clean results of the type we have

obtained for balanced truncation. We will be looking for simplified problems with more analytic properties as well as studying computationally oriented approaches to the general problems.

2.1.4 Modeling from first principles

We plan to continue our program of extending the LFT (Linear Fractional Transformation) and μ framework [25] to modeling and model validation, and ultimately to system identification [71, 55]. Our greatest success to date in extending the LFT/ μ framework to make more direct and systematic contact with modeling is in the area of control of flexible structures [3, 53, 52]. Here we developed general methods for modeling the interconnection of uncertain models of substructures, but identification to obtain initial models remains fairly *ad hoc*.

While details of modeling must necessarily remain domain specific, we believe it is possible to provide a general framework for modeling engineering systems that is more amenable to control design. For example, rather than accepting models of physical systems in terms of, say, linear differential equations, the control engineer should encourage the use of LFTs with explicit representations for those uncertainties known to be present on the basis of first-principle modeling. Also, the LFT concepts for modeling uncertainty should be extended to nonlinear systems to increase their flexibility. Although extending all the μ machinery for analysis of nonlinear systems is very long-term research problem, the basic manipulation of *uncertain* nonlinear models in a manner similar to that available for LFT models should be reasonably straightforward. The deeper questions of how to do this in a way that is both natural to engineers and facilitates the subsequent analysis will be one focus of this research program.

2.1.5 Computational Issues in Solving LMIs

One of the most attractive features of the new results is the central role played by LMIs. Since LMIs are linear and convex, they are potentially computationally attractive. Although general purpose convex optimization routines will provide polynomial time algorithms, substantial improvements should be possible by exploiting the special structure of LMIs. This is something we are currently investigating [9].

2.2 Analysis

2.2.1 Introduction to the μ Framework

The canonical μ framework is shown in Figure 3. The system under consideration is represented by M , and Δ is an uncertainty description. This uncertainty description may be block structured, and may include repeated parameters. Note that the most familiar use of μ involves treating the uncertainties as norm bounded LTI perturbations (e. g. unmodeled dynamics) or equivalently as complex uncertainties at any given frequency. A number of different extensions to these concepts will be outlined in the following sections of this report. Note also that this setup is quite general. Any interconnection of systems and perturbations can always be rearranged into this canonical framework, with Δ as a block structured perturbation, and M as the system matrix resulting from the interconnection.

The most well known use of μ as a robustness measure is in the frequency domain. Suppose $M(s)$ is a stable transfer matrix of an LTI system. Further suppose that M is partitioned as:

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

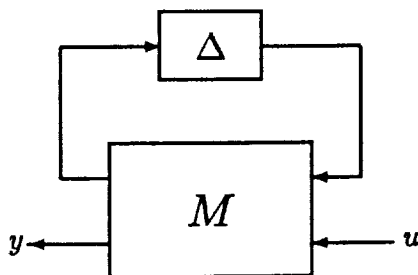


Figure 3: Feedback interconnection for μ analysis

Then μ is essentially defined as the answer to the robust stability problem in the following sense. The interconnection in Figure 3 is internally stable for all Δ 's norm bounded by one if and only if $\mu(M_{11}) \leq 1$ for all frequency. Furthermore if our performance criterion is that the worst case L_2 - L_2 gain from u to y should be less than one (i. e. an H_∞ performance criterion) then we have robust performance if and only if $\mu(M) \leq 1$ for all frequency. For a more rigorous definition of these concepts see [28, 29].

These results mean that we can evaluate the robustness properties of our closed loop system, by using a frequency evaluation of μ . Note that at any given frequency point we have a constant matrix μ problem, for which good upper and lower bounds have been developed (see [28, 60]) and commercial software is available [5]. This traditional use of μ , which has been widely used for robustness analysis and synthesis, enables us to consider robustness of LTI systems to LTI perturbations. In this report the canonical μ framework outlined above is used as the starting point for a number of new directions, which are aimed at developing much more powerful analysis and synthesis methodologies.

2.2.2 Mixed μ Analysis

In recent years a great deal of interest has arisen with regard to robustness problems involving parametric uncertainty. These parameters can represent coefficients in a model which have a natural physical interpretation, such as masses, aerodynamic coefficients, etc., and are only known to lie within some range. Hence we obtain problems involving uncertain parameters that are not only norm bounded, but also constrained to be real. Of course we still wish to allow complex uncertainties in our robustness problems, to cover unmodeled dynamics, and to allow us to handle both robust stability and performance questions. This type of robustness problem, involving both real and complex structured uncertainty, can be treated within a unified framework by formulating a *mixed* μ problem, where the block structured uncertainty description is now allowed to contain *both* real and complex blocks. This mixed μ problem can have fundamentally different properties from the more familiar complex μ problem (where the block structured uncertainty description contains only complex blocks), and these properties have important implications for both the theoretical and computational aspects of the problem. In this section we give a brief review of some recent results in this area, and some of the current research directions being pursued at Caltech on mixed μ problems (see [28, 29, 58, 30, 60] and the references therein for results pertaining to the complex μ problem).

Having cast our robustness problems in the mixed μ framework, the analysis question (for robust

stability and/or performance) reduces to one of evaluating mixed μ . This is an area where much recent progress has been made, and continues to be an area of active research, as is outlined in the following subsections.

Fundamental Properties

It is now well known that real μ problems can be discontinuous in the problem data (see [8]). As well as adding computational difficulties to the problem this sheds serious doubt on the usefulness of real μ as a robustness measure in such cases, since the system model is always a mathematical abstraction from the real world, and is computed to finite precision. However it is shown in [59] that mixed μ problems containing some complex uncertainty are, under some mild assumptions, continuous in the problem data (whereas purely real μ problems are not). This is reassuring from an engineering viewpoint since one is usually interested in robust performance problems (which therefore contain at least one complex block), or robust stability problems with some unmodeled dynamics, which are naturally covered with complex uncertainty. Thus in problems of engineering interest, the potential discontinuity of mixed μ should not arise.

Recent results in [65] show that a special case of computing μ with real perturbations only is NP complete. While these results do not apply to the complex only case, new results in [12] show that the general mixed problem is NP complete as well. These results strongly suggest that it is futile to pursue exact methods for computing μ in the purely real or mixed case for even moderate (less than 100) numbers of real perturbations, unless one is prepared not only to solve the real μ problem but also to make fundamental contributions to the theory of computational complexity. Furthermore, it may be that even approximate methods must have worst-case combinatoric complexity [22].

Upper and Lower Bounds

The above results do not mean, however, that "practical" algorithms are not possible, where "practical" means avoiding combinatoric (nonpolynomial) growth in computation with the number of parameters for all of the problems which arise in engineering applications. Practical algorithms for other NP hard problems exist and typically involve approximation, heuristics, branch-and-bound, or local search. Results presented in [80] strongly suggest that an intelligent combination of all these techniques can yield a practical algorithm for the mixed problem.

Upper and lower bounds for mixed μ have recently been developed, and they take the form of generalizations of the standard bounds for the complex μ problem [28, 60] (i.e. by applying the mixed μ bounds to complex μ problems one recovers the standard complex μ bounds). The upper bound was presented in [31] and involves minimizing the eigenvalues of a Hermitian matrix. This can also be recast as a singular value minimization which involves additional scaling parameters to the complex μ upper bound. It is shown in [79] that the mixed μ problem can be recast as a real eigenvalue maximization and that this in turn can be tackled via a power algorithm, giving a lower bound for mixed μ . A practical computation scheme for these bounds has recently been developed [81] and will be available shortly in a test version in conjunction with the μ -Tools toolbox [5].

The quality of these bounds, and their computational requirements as a function of problem size, are explored in [80]. It is seen that the computational requirements are reasonable for up to medium size problems (less than 100 perturbations). While the bounds are usually accurate enough for engineering purposes, in a significant number of cases of interest, they are not. This is in contrast with the purely complex nonrepeated case, where no examples of problems with large gaps have been found. The use of Branch and Bound schemes to improve upon existing bounds has been suggested by several authors (see [2, 70] and references therein). There are some important

issues and tradeoffs to be considered in implementing such a scheme, which can greatly impact the performance. A selection of results from a fairly extensive numerical study of these issues is presented in [80], and a Branch and Bound scheme is proposed which should form the basis of a practical computation scheme for mixed μ . This will be further explored in [56].

The upper and lower bounds from complex μ theory not only serve as computational schemes, but are theoretically rich as well. Connections between the bounds and various aspects of linear system theory have been established, and further work in this area appears to have great promise. A theoretical study of the mixed μ bounds may yield new insight as well, and this is a subject of current research. Initial results in this area are presented in [78], where it is seen that mixed μ inherits many of the (appropriately generalized) properties of complex μ , although as has already been seen, in some aspects the mixed μ problem can be fundamentally different from the complex μ problem.

The Rank One Case and "Kharitonov-Type" Analysis

Problems involving robustness properties of polynomials with coefficients perturbed by real parameters have received a great deal of attention in the literature. This type of robustness problem leads to a (real or) mixed μ problem. Several celebrated "Kharitonov-type" results have been proven for special cases of this problem, such as the "affine parameter variation" problem (see [7] for example), and the solutions typically involve checking the edges or vertices of some polytope in the parameter space. It can be shown that restricting the allowed perturbation dependence to be affine leads to a real μ problem on a transfer matrix which is rank one. Note that this "rank one" assumption is very restrictive. Typically robustness problems motivated by real physical systems do not satisfy this assumption.

The rank one mixed μ problem is studied in detail in [20]. The authors develop an analytic expression for the solution to this problem, which is not only easy to compute, but has sublinear growth in the problem size. They are then able to solve several problems from the literature, noting that these problems can be treated as special cases of "rank one μ problems" and are thus "relatively easy to solve". Even the need to check (a combinatoric number of) edges is shown to be unnecessary.

This rank one case is also studied in [77], where it is shown that for such problems μ equals its upper bound and is hence equivalent to a convex problem. This reinforces the results of [20] and offers some insight into why the problem becomes so much more difficult when we move away from the "affine parameter variation" case to the "multilinear" or "polynomial" cases [70]. These correspond to μ problems which are not necessarily rank one, and hence may no longer be equal to the upper bound and so may no longer be equivalent to a convex problem.

These results underline why there are no practical algorithms based on "edge-type" theorems, as the results appear to be relevant only to a very special problem. Furthermore, even in the very special "affine parameter case" there are a combinatoric number of edges to check.

Practical Applications

The upper and lower bounds discussed in the preceding subsections have been implemented in software. This software is currently being β -tested at several industrial and academic sites, including Honeywell, Phillips, NASA Dryden, Caltech, U. C. Berkeley and others. A test version is scheduled for commercial release in September 1992, in conjunction with the μ -Tools toolbox [5].

A number of interesting applications of the software to problems arising from real physical systems have already been undertaken. The control design of a missile autopilot is considered

in [4]. The software is used to examine the robustness (in performance) of the control design to perturbations in Mach number, angle of attack, and unmodeled dynamics. This results in a mixed μ problem with two repeated scalar real parameters and three full complex blocks. The mixed bounds were found to be quite different to the bounds one would obtain from the associated complex μ problem, and the performance predictions were borne out by the simulations.

Control of a flexible structure is considered in [6], and the robustness of the design is evaluated with respect to variations in the natural frequencies of the structural modes, as well as unmodeled dynamics. This results in a mixed μ problem with five scalar real parameters and three full complex blocks. Interestingly in this case, because of the way the uncertainties entered the system, the mixed and complex bounds were found to be very close. The control designs were verified in simulation and experiment. For these (and several other) examples the software worked well, providing tight bounds for the associated mixed μ problems.

The Next Generation of Algorithms

A number of improvements to the present computation schemes are under development. One research direction is to improve the algorithms for computing the bounds. This is being actively pursued at Caltech, and we refer the reader to [73] for the use of adaptive power iteration to improve the lower bound performance, and [10] for the use of LMI techniques to improve the upper bound computation.

Note however that the bounds from [31, 79] may be far apart (regardless of the computation method). For these cases one must consider improving the bounds themselves. A promising approach is to use the existing bounds as part of a Branch and Bound scheme, which iteratively refines them. In this way one can develop a scheme to compute *guaranteed* bounds for the mixed μ problem. Since the problem is NP hard one must expect that the worst case computation time for such a scheme will be exponential. The real issue is whether or not one can produce a "practical" scheme, whose *typical* computation time is polynomial. We believe that it is possible to develop such a scheme, using the results from [80], and this will be further pursued in [56].

2.2.3 LPV μ (\mathcal{Q} Stability)

Linear Parameter Varying μ refers to a recent generalization of the standard structured singular value setup. This generalization tests robustness and robust performance for time varying and nonlinear perturbations. In this context, the uncertainties are operators from L_2 to L_2 with induced norm bounded by one. Linear parameter varying μ gets its name from the useful interpretation of perturbations as unknown varying parameters, as in the case of θ below.

$$\begin{aligned}x_{k+1} &= A(\theta(k))x_k + B(\theta(k))u_k \\y_k &= C(\theta(k))x_k + D(\theta(k))u_k\end{aligned}$$

To clarify the difference between LPV μ and LTI μ we must first review a few features of LTI μ . The standard (LTI) μ software in general use typically calculates bounds for μ by doing a "fresp", that is a frequency response plot. The fresp is an attempt to calculate $\sup_w(\mu(M(w)))$ which is equivalent to a single μ test. Figure 4 represents this equivalence. The principle reason for doing the fresp is that the one-shot formulation typically yields large gaps between the upper and lower bounds for μ , whereas the fresp version typically generates a tighter bounds for μ .

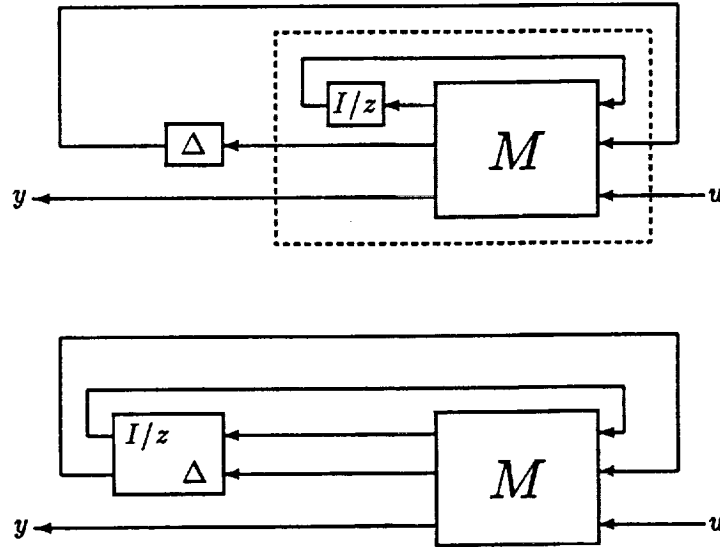


Figure 4: Frequency Domain μ (top) and State Space μ (bottom).

The LPV μ test is also a one-shot test, but there is no fresp version of the test. An obvious reason for this is that there is no frequency domain for time varying or nonlinear operators. This is of no consequence because of the important recent result [69] that LPV μ is equal to the one-shot upper bound for LTI μ . In light of this, it is quite reasonable that there can be large gaps between LTI μ and its one-shot upper bound.

The one-shot upper bound for LTI μ (which is equal to LPV μ), commonly referred to as \mathcal{Q} stability margin [44], is a convex optimization problem, and is consequently computationally tractable. In addition to the standard, commercially available, software to compute the upper bound for LTI μ , there are optimization schemes which give an indication of how well the optimization is doing.

In addition to robustness analysis, there is a robust performance version of LPV μ just as there is for LTI μ . Furthermore, controller synthesis is easier for LPV μ than for LTI μ . At first glance, LPV μ looks like a very useful tool for the controls engineer. However, there are important limitations to LPV μ . LPV μ is a necessary and sufficient test for robust stability or robust performance of an LTI system in the presence of norm bounded, but otherwise arbitrary, perturbations or uncertainties. It is the worst case perturbations that determine stability and performance. Consequently the analysis may be extremely conservative for perturbations that are not worst case. In practice there are many time variations or nonlinearities that may be known, or be constrained to be slowly varying, and we would like a nonconservative analysis technique for such cases. LPV μ does not generally prove adequate for such cases except when LPV μ is close to LTI μ , that is except when time variations don't make much difference.

Currently, LPV μ is only understood for the case of scalar uncertainty blocks. We expect that results for more cases, *e.g.* repeated scalar uncertainty blocks, will be forthcoming.

2.2.4 Frequency Domain Consistency Analysis

Robust control theory poses a problem of deciding whether a model is suitable for design and analysis. There is a rich class of models to choose from, and it is difficult even to tell which ones are consistent with experimental data. The consistency analysis problem (also referred to as the model validation problem) in its simplest form is: given experimental data and a model with both additive noise and norm-bounded perturbations, is it possible that the model could produce the observed input/output data?

Consistency analysis can be reformulated as a generalization of μ [55]. The $\rho(QM)$ lower bound to μ is generalized to a lower bound for the generalization of μ . Computation of this lower bound can be attacked using a generalization of the power algorithms used for standard μ . The natural upper bound for the generalization of μ is not a generalization of $\bar{\sigma}(DM D^{-1})$. Rather, the upper bound is formulated as an LMI where the solution matrices are still structured, but are no longer all positive definite. Instead, some blocks of the solution matrix must be positive definite, while other blocks must be negative definite.

In addition to providing a connection between real data and robust control models for LTI systems, model consistency is the first instance we encounter of a connection between robust uncertainty models and a *prescribed* time variation or signal, rather than a worst case variation or signal. Recall that LTI μ considers only worst case inputs and worst case LTI perturbations, while LPV μ is similar except that the worst case perturbation may be time varying or nonlinear. In both cases there is a single μ test that answers the problem. In contrast, the model consistency problem requires the solution to many μ type problems. This seems natural when we consider that collecting more data should require more consistency checking. Note that nonlinear simulation does provide for some prescribed variation, but there is no uncertainty analysis beyond running the simulation for *prescribed* perturbations to the nominal system, whereas μ tests check all the perturbations at once.

An important approximation the frequency domain model consistency analysis makes is that there is a frequency domain. For finite horizon signals, as opposed to periodic signals, there is no frequency domain. The benefit of this approximation is that it allows the consistency problem to be decoupled into independent μ type problems at each frequency. The resulting tests are similar to an LTI μ fresp, but in the consistency case there is no one-shot alternative to the fresp. In the time domain tests discussed next, we will see that not only is there no simple one-shot test, but also that the problems don't decouple the way they do in the case of frequency domain consistency analysis. Rather, the time domain problems are coupled together in one very large μ type problem, giving rise to the moniker "huge μ ".

2.2.5 Time Domain μ analysis.

A crucial step in the generalization of robust control theory to time varying and nonlinear problems as well as to robust identification problems is the development of time domain μ analysis. To understand the time domain μ setup, we first consider a simple time domain interconnection. Figure 5a shows two steps of a discrete time linear time varying system, while Figure 5b shows the same interconnection arranged in a different form. We emphasize that the operators from (x_0, u_1, u_2) to (x_2, y_1, y_2) in the the two diagrams are identical. Note that this rearrangement is easy for any number of time steps.

It is easy to form the interconnection of two time steps of an uncertain time domain operator shown in Figure 6a and to form the equivalent system shown in Figure 6b. Again, it is easy to generalize this to any number of time steps. Figure 6b is very reminiscent of the standard μ analysis

setup. Once we define an appropriate notion of performance from input u to output y and from initial condition x_0 to final state x_n (where $n = 2$ in the figure), then we can add a performance block to the uncertainty structure in Figure 6b. This results in a robust performance μ problem for a finite horizon discrete time (possibly time varying) uncertain system. This application of μ is called linear time varying μ or LTV μ .

It is important to note that the analysis is for worst case performance for the worst inputs, initial conditions, and uncertainties, and for the *prescribed* time variation in M . This is in marked contrast to the LPV μ case, where there are no prescribed time variations, only worst case time variations.

This formulation of LTV μ would not be of much interest without some hope of a computational algorithm that takes advantage of the special structure of the problem (e.g. the special structure of the matrix M in Figure 6b). Current μ computation time for unstructured matrices typically grows with n^3 , where n is the size of the problem. Consequently, computation time of current algorithms for the LTV μ problem would grow with the number of time steps cubed, and would thus be impractical for many problems of interest. To see how an efficient algorithm might be possible, observe that Figure 6b can be rewritten as Figure 7 where P_l and P_r are permutation matrices (their output is simply a reordering of the rows of their input). This structure admits a modification of the lower bound power algorithm for μ computation that is both very easy and very efficient. We expect the upper bound calculation also to benefit substantially from this special structure.

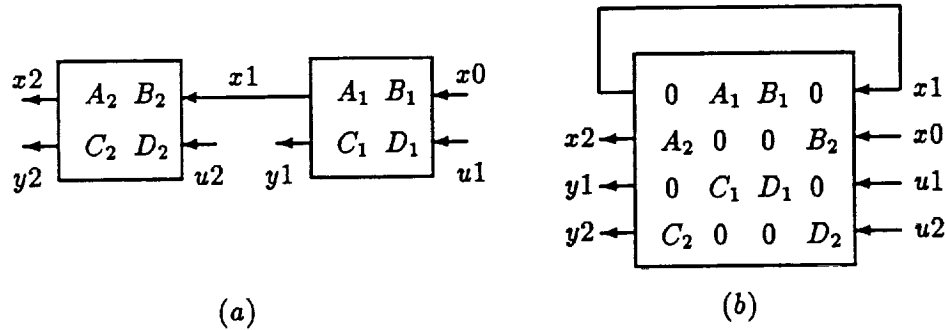


Figure 5: Simulation as a matrix operation.

2.2.6 Time Domain Mixed μ Analysis

If we allow the further constraint that some uncertainty blocks be real, time domain μ analysis becomes time domain mixed μ analysis. Computation of the upper bound for time domain mixed μ analysis, while a straightforward combination of the time invariant mixed μ upper bound and the special structure of the time domain complex μ problem, will require some careful investigation, as the mixed μ upper bound computation is rather intricate.

A very important special case of time domain mixed μ is easy. When all the uncertainties are constrained to be real, and the time varying plant and its initial condition are real, then the worst case solution is all real. In this case the computation does not require the added complexity of the mixed μ computation.

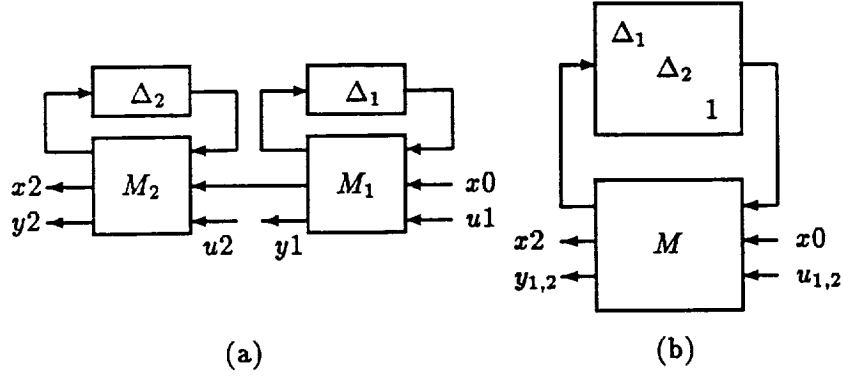


Figure 6: Simulation with model uncertainty

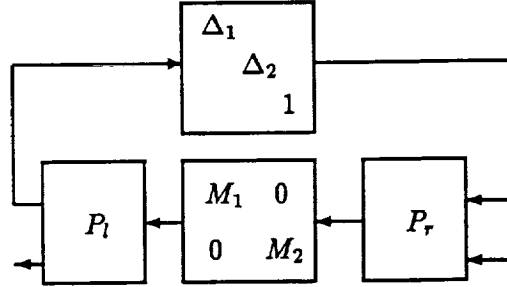


Figure 7: Computation setup

2.2.7 Time Domain Consistency Analysis

As in the case of time domain mixed μ analysis, time domain consistency analysis requires an extension of an algorithm for a general matrix to efficiently handle a much larger and highly structured matrix. While the lower bound should extend easily, the upper bound may be problematic. The lower bound for consistency analysis is an extension of the lower bound power algorithm for mixed μ , and can easily take advantage of the structure of a time domain problem the same way the lower bound for time domain μ does. The upper bound for consistency analysis, however, is not an extension of the upper bound for mixed μ analysis. Instead, the upper bound for consistency analysis requires an LMI formulation. LMI computation is a hot topic in the field now, and it is anticipated that research there will yield a practical upper bound computation for frequency domain consistency analysis. The resulting upper bound for frequency domain consistency analysis must then be modified for the time domain consistency analysis to take advantage of the special structure of the time domain problem.

2.2.8 μ on Trajectories

A nonlinear simulation results in a trajectory. If we linearize the nonlinear system about this trajectory, we get a linear time varying system. Since time domain mixed μ will allow us to analyze robust performance for linear time varying systems, we will seek to extend this analysis to one

appropriate for nonlinear systems along a trajectory. (Currently, robustness analysis for nonlinear systems consists of many simulations for various prescribed perturbations from the nominal.) The difficulty here is that this extension appears to require that the (very large) matrix for which we are computing μ now depends on the solution to the μ computation. One approach to this difficulty is to iterate, using the new trajectory as the nominal trajectory for the subsequent iteration. This approach might be substantially improved by incorporating some sort of trajectory iteration with each iteration within the μ computation. If this proves successful, the resulting analysis would replace the tedious and endless simulation typically required for nonlinear robustness analysis.

2.2.9 Robust Identification

Robust identification attempts to find parameter values and uncertainty bounds, for robust control models, that best describe the observed behavior of the system. Currently, robust identification is entirely ad-hoc, proceeding without even any consistency checks. We propose to develop an algorithm that finds optimal parameter values that are consistent with observed data. This will be done via an extension of the time domain consistency analysis. In [72] a very similar problem is posed in the frequency domain.

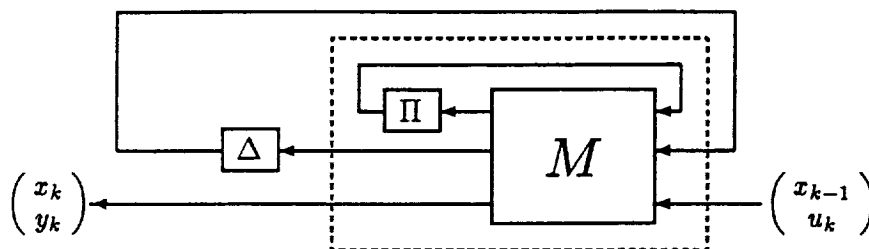


Figure 8: LFT Parameter Dependence

We consider models that are an LFT on some parameters we wish to identify. Then the single time step building blocks for time domain consistency analysis are as in Figure 8. When the separate single time step building blocks are combined together we get the setup for robust identification. This setup is different from the time domain consistency analysis in that the uncertainty structure contains blocks with the as yet unknown parameter values. Typically, we have constraints on the parameter values we wish to identify. In this case the necessary modifications of the time domain consistency analysis algorithm are easy and are analogous to modifications of standard algorithms already being pursued in other research.

2.2.10 Nonlinear Scheduled Identification

Our ultimate goal for an identification technique is a combination of robust identification and μ along trajectories. Any sort of identification requires a priori knowledge or assumptions. Typical assumptions include causality, time invariance, minimum decay rate of (possibly time varying) impulse response etc. When we consider the case of nonlinear identification, the role of a priori knowledge is crucial: any collection of input/output data is consistent with a collection of nonlinear operators that is so large that they collectively have no consistent predictive value. Furthermore, individual records of input/output data in general do not contain information about the global behavior of nonlinear systems, instead they contain information of behavior along a trajectory.

Consequently we expect that any nonlinear identification scheme of general interest must contain the features inherent in the combination of robust identification and μ along trajectories.

2.3 Synthesis

2.3.1 Mixed μ Synthesis

The problem of synthesizing a controller which is (optimally) robust to structured mixed uncertainty is very difficult, since the associated optimization problem is not convex. Some exact solutions have been presented for special cases of this problem (see [63] for example, which reduces the "rank one" μ synthesis problem to a convex optimization problem), but as yet there is no globally optimal solution to the general problem (even in the purely complex case).

Despite this fact the (complex) μ -synthesis procedure first outlined in [29] has been successfully applied to a large number of engineering problems (see [6] for example). This procedure involves a "D-K iteration" between computing the μ upper bound, and solving for an H_∞ (sub) optimal controller (both of which are convex problems). This procedure, which was developed for μ problems involving *only* full complex blocks, does not guarantee to find the globally μ -optimal controller, but has often been found to work well in practice.

One of the current research goals at Caltech is to extend this procedure to the mixed case (initially for *nonrepeated* real scalars and full complex blocks), by exploiting the new analysis tools for the mixed μ upper bound described in the preceding subsections. Substantial progress has already been made on this problem, and prototype software for mixed μ synthesis is currently under development. In principle one can further extend this procedure to allow for repeated (real or complex) scalar blocks as well. In order to do this however, one needs to be able to fit a MIMO transfer matrix to frequency response data (for nonrepeated blocks only SISO fits are required), and a number of practical issues need to be worked out before an efficient implementation of this procedure can be developed for the repeated case.

2.3.2 Stabilization of LPV/LFT systems

The key breakthrough in LPV/LFT synthesis was a series of results and machinery initiated by Lu, Zhou, and Doyle in [44]. In this paper, necessary and sufficient conditions are derived for when an LFT system can be stabilized via an output feedback controller which depends on the same Δ as the plant. Both μ and \mathcal{Q} stability are considered, and a direct generalization of the Youla parametrization of all stabilizing controllers is obtained via a general separation theorem that reduces the output feedback problem to Full Information (FI) and Full Control (FC) problems. For \mathcal{Q} stability the FI and FC problems can be solved iff a certain LMI is satisfied, and the stabilizing controller can always be taken to be a constant gain.

The machinery developed in [44] and subsequent research by Lu and Doyle is perhaps more important than the specific theorems. This machinery allows conventional stabilization theory involving observers and state feedback to be extended to a much larger class of uncertain and time-varying systems. These stabilization results have been further extended recently in a number of directions. For example, Packard and Becker [57] consider stabilization of LPV systems with parameters that need not enter as LFTs and also address computational issues. There are also extensions to optimal control problems in recent work, primarily due to Packard, Zhou, Doyle and their coworkers. They address three control problems, all of which involve reformulation into LMIs. The problems are:

- Gain-scheduled, state-feedback with \mathcal{Q} stability criteria for uncertain systems.

- Optimal, constant, block diagonal, similarity scaling for full information and state feedback H_∞ problems.
- A theory for optimal performance in systems which depend on several independent variables. It provides a new and much simplified development of the state-space H_∞ theory and a direct generalization to multidimensional systems and gain-scheduled controllers for uncertain systems.

The use of the term “gain-scheduled” may be a little misleading as the usual use of this term implies parameters that are slowly varying, whereas most of the theories described above neither requires nor exploits that. Another line of work involves mixed H_2/H_∞ optimal control, which directly generalizes the standard H_2 and H_∞ theories. The most immediate research question has to do with the engineering implications of these new results. Since the new stabilization results and these optimal control results have a certain gain-scheduling interpretation, we will compare them with other results on gain scheduling.

2.3.3 Connections between LPV and LTV systems

An obvious question, and one that has great potential for fruitful research, considers the relationship between the recent work in LPV stabilization and standard methods for LTV systems. Since the publication of [23], there have been numerous extensions of the state-space H_∞ methods to LTV systems. We will compare robust designs for LFTs with unknown but time-varying perturbations with optimal time-varying controllers for particular time-varying values of the perturbations.

2.3.4 “Gain Scheduling” by LPV stabilization and μ synthesis

We consider two distinct scheduling schemes which use standard μ synthesis D-K iteration, but in slightly different ways. One method allows the controller to be an LFT on the scheduling parameters θ . Such a setup is shown in Figure 9(a), and is equivalent to the rearranged system in Figure 9(b). Note that since θ now enters K only through P' , Figure 9(b) is a standard mixed μ synthesis problem. The resulting design is robust for the worst case constant θ , and retains some robustness for slowly varying θ . Note that this problem requires mixed- μ synthesis with repeated parameters, which is a subject of current research.

Another method is motivated by recent results from [44], where the authors develop a parametrization of all \mathcal{Q} stabilizing controllers which vary with the parameters θ . These controllers are parametrized in terms of a stable free (possibly time varying) parameter Q . The scheduling procedure is as follows. For each θ_k in a set, design a (stable) Q_k for the “frozen system”. Then choose suitable $\lambda_k(\theta)$ to define an interpolated $Q(\theta) \doteq \sum_k \lambda_k(\theta) Q_k$. As $Q(\theta)$ is stable, the resulting closed loop system is guaranteed to be stable (even for rapidly varying parameters), and much of the “frozen system” performance characteristics are retained for slowly varying parameters.

Both of the techniques suggested need to be much more systematic. We plan to develop a single methodology that includes both methods as special cases and eliminates several of the *ad hoc* choices.

2.3.5 LTV and LPV/LTV μ synthesis

This work will seek to generalize the D-K iteration of μ synthesis by using standard LTV H_∞ for the K part. The choice of the D is not so obvious, as there is no natural separation of the problem in the frequency domain. We expect to use the time-domain μ analysis on finite horizons

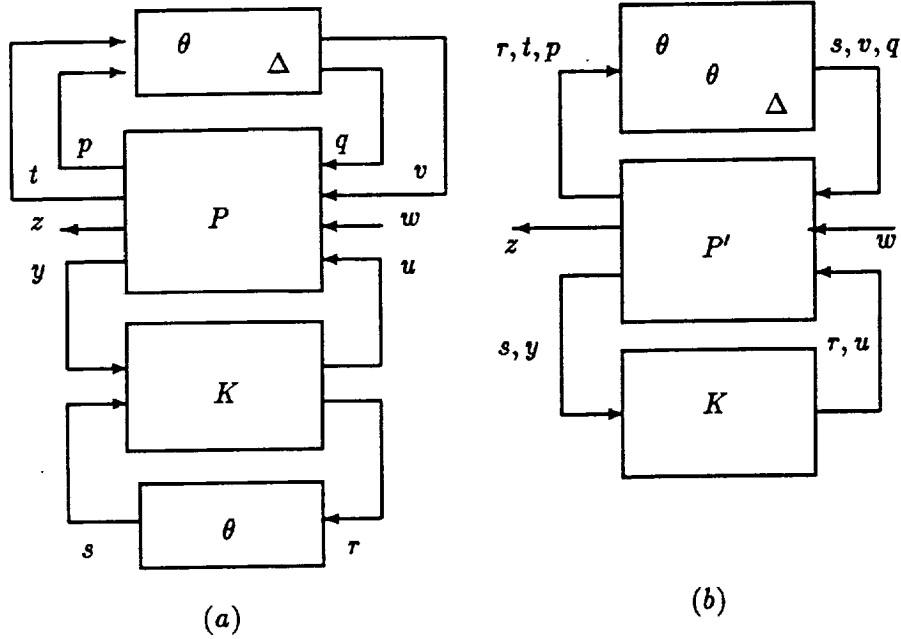


Figure 9: Synthesis setup for slowly varying parameters

to select suitable D scalings. This will be extended to mixed real and complex time-domain μ as that becomes available. Combined LPV/LTV μ synthesis will then build on connections established between LPV and LTV theories.

2.4 Nonlinear control theory

It is a general principle for nonlinear systems that properties of the linearization of the system about a point can be extended locally to the full nonlinear system. If the linearization of a nonlinear system is controllable, control laws for stabilization and tracking of the linearization can be used for *local* stabilization and tracking of the original system. Often the regions in which these control laws can be applied is quite small relative to the desired operating region of the system. The challenge of nonlinear control theory is to design controllers which satisfy design criteria in larger operating regions and provide improved performance as compared to a linear control design.

2.4.1 Feedback linearization and nonlinear inversion

One powerful approach to increasing stability regions for nonlinear systems is to search for a feedback transformation which converts the nonlinear system into a linear one. That is, we search for a coordinate transformation and a state-dependent precompensator such that the resulting system, in the new coordinates, is linear. If such a transformation is possible, the system can be stabilized on the region in which the feedback transformation is well-defined using linear methods. For some classes of systems, such as robot manipulators, the transformation is defined everywhere and global stabilization of the nonlinear system is possible. The literature on this subject is vast; see [38] for an introduction to the basic concepts.

The necessary and sufficient conditions for feedback linearization require that the linearization

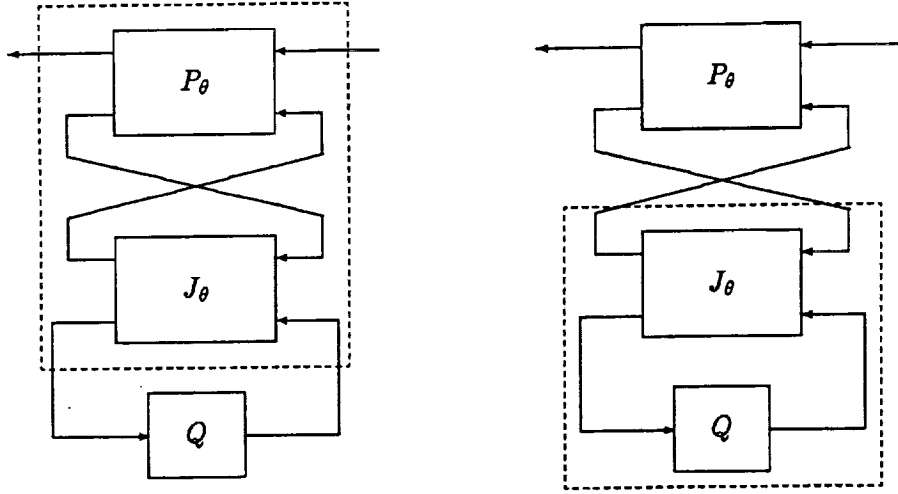


Figure 10: $P_\theta - J_\theta - Q$ interconnection

of the system be controllable and that a certain set of vector fields form an involutive distribution. The involutivity condition is the more restrictive of the two, and is not satisfied for many important nonlinear systems. Systems which cannot be feedback linearized via static state feedback can sometimes be linearized using dynamic precompensators. The basic idea is to add integrators to selected input channels to achieve a system which can be I/O decoupled and linearized. Both static and dynamic feedback linearization require measurement of the states for use by the precompensator. Some results are available for linearization via output feedback [46], but the conditions under which such a feedback exists become considerably more restrictive than the full-state feedback case.

A serious drawback of all feedback linearization techniques is the failure to account for uncertainties. As with many inversion methods, feedback linearization can be extremely non-robust. The effect of modeling errors is understood in some simple situations (where so-called “matching conditions” are satisfied), but a general framework for analyzing the effects of uncertainty on the performance and even stability of the controller is not available.

More significantly, the disturbance rejection properties of nonlinear controllers can be extremely poor. As a simple example, consider the scalar nonlinear system

$$\begin{aligned}\dot{x} &= e^x(u + d) \\ y &= x + n\end{aligned}$$

where u is the control input, d models actuator noise, n models sensor noise, and y is the plant output. The precompensator $u = e^{-x}v$ feedback linearizes the system when the disturbances are ignored. The usual approach is to now design a controller for the linearized system. If we seek to minimize the H_∞ gain between the disturbance vector (d, n) and the output (y, v) for the linearized system, the optimal feedback is $v = -x$. However, with this choice of precompensator and feedback, the closed loop system becomes

$$\dot{x} = -x + e^x d$$

which has finite escape time for a disturbance $d > e^{-1}$. In fact, for this particular example system, it can be shown, using either operator theoretic techniques or nonlinear H_∞ techniques, that the optimal H_∞ gain for the full nonlinear system is $\sqrt{2}$ and that this can be achieved using unity feedback. This example illustrates that blind application of linearizing feedback can give extremely poor performance.

Additional results are available when the control specification is to track a desired output signal rather than regulate the system to a point. These results include tracking the output of a model (linear) system by solving a model matching problem [11] and tracking the output of an undriven exosystem using nonlinear regulator theory [39]. Necessary and sufficient conditions for solving these problems are known and, as in the exact linearization case, are highly restrictive. An additional requirement is that the system remain in the neighborhood of a fixed equilibrium point.

Related to linearization techniques, are so-called "dynamic inversion" approaches, which have found recent application in aircraft flight control [76, 51, 13]. In dynamic inversion, a state-dependent input coupling matrix is inverted and nonlinear terms in the dynamics are directly cancelled. The resulting system is then controlled via linear control techniques, often using a nominal state-space trajectory which is calculated a priori using a simplified approximation of the nonlinear system. The work in this area is reminiscent of computed torque control of robot manipulators, where dynamic nonlinearities are cancelled before applying feedback control. As with more general nonlinear control techniques, research into the disturbance rejection and robustness properties of this class of controllers is still needed.

2.4.2 Approximate methods

To extend the methods for nonlinear stabilization and tracking to a larger class of systems, it is advantageous to study approximation of nonlinear systems. The basic idea is to approximate a given nonlinear system by one which satisfies the restrictive conditions necessary to design a nonlinear controller. The simplest (and most classical) such approximation is the linearization of a system about a constant operating point. By designing an appropriate control for the approximate system, we achieve a controller which works locally for the nonlinear system. The goals in using more complicated approximations are to extend the region in which a controller can operate and to improve performance.

One technique for building controllers is through the use of approximate feedback linearization. In this method we construct a system which approximates the plant in some appropriate sense and which is also feedback linearizable. We then proceed to design a controller for the approximate system and apply it to the original system. This technique often results in a system which is controlled in a larger region of an equilibrium point [37], or, in the case of a uniform system approximation, in a region about an entire equilibrium manifold [35]. Typically a "slowly-varying" condition is required which limits the magnitude and speed of the reference trajectory. These techniques generalize linearization about a point, and, in the latter case, are related to gain-scheduling methods. Since we allow higher order approximations, we expect to be able to find controllers which perform better than their linear counterparts. These results are at least partially verified using a robotic model of an acrobat (dubbed the *acrobot*) as a simple example [36].

A related approach has been used by Krener to construct polynomial approximations to nonlinear systems [41, 40]. Using ideas based on Poincaré linearizations of nonlinear systems, one can construct approximations of a given order which are feedback linearizable. Again we assume that activity occurs near a single operating point and hence we can search for a single linear controller which provides adequate performance. Extensions to the case of motion near an equilibrium

manifold would appear to present no major difficulties. The primary limitations are the use of Taylor series (which limit the range in which the approximation is valid) and satisfaction of a linear resonance condition.

It is also possible to extend output regulation techniques using approximate mappings between the exosystem and plant trajectories. As in the approximate feedback linearization, the basic result using this technique is that trajectories which are slowly varying and close to an equilibrium point can be approximately tracked.

A common theme in all of these approximation methods is the need to remain near equilibrium points of the system. In addition, it is also necessary to limit the speed of the system and hence the controllers only work in situations where slowly-varying outputs must be tracked. Current research is searching for methods to generate approximations which provide a coarser approximation to the system, but in a larger region. With such an approximation, it may be possible to control the global behavior of the system far away from equilibrium points and then switch to a more precise local controller when the system is operating near a fixed operating point.

A feature of the approximate linearization techniques described above is a large degree of freedom in constructing the approximate vector fields which are used to generate a controller. Currently there is little understanding of how to choose between different approximations so as to improve the performance of the overall controller. One of the goals of this research effort is to develop an understanding of how the choice of different approximations affects the overall control design process and, more importantly, how the overall control design process can be used to optimize the choice of approximations.

2.4.3 Model predictive control

A system's operation is always subject to constraints, i.e., nonlinear elements which the controller must be equipped to handle. Most common are actuator saturation constraints but other operating constraints are usually present as well. Over the last decade Model Predictive Control (MPC), also referred to as Receding Horizon Control, has emerged as the technique of choice for dealing with complex constraints. MPC has attracted academic attention [48, 47, 49, 61, 62] and has been adopted widely in the process industry [21, 33].

In the model predictive control formulation, the control objective is expressed in terms of a single (usually quadratic) objective function and the operating constraints are translated into (usually linear) inequality constraints. The algorithm involves the on-line solution of a constrained optimization problem to determine a set of piecewise constant (discrete), feasible future inputs which will cause predicted values of future plant outputs to track a prescribed trajectory "as closely as possible" for a specified time period (horizon) into the future. Feasible future inputs are those which do not violate any input constraints and produce predicted outputs which do not violate any output constraints. This optimization is solved using mathematical programming techniques. Although several future input values are calculated at each sampling time only the first control action is implemented and then the horizon is moved forward ("moving horizon" or "receding horizon") and the "optimal" control inputs are recalculated based on the updated information (measurements) about the system.

Because of the predictive nature of the algorithm, constraint encounters are anticipated and the control action is smooth. This soft compromising behavior is undesirable if there are several constraints with clear priorities. The main tuning parameters are the weights on the different terms in the objective function and the length of the horizon. Because a mathematical program is an integral part of the control algorithm it is very difficult to study its stability, performance and

robustness analytically. Therefore, the tuning parameters are mostly selected by trial and error based on simulations. For the tuning the behavior of the system has to be considered when it is unconstrained as well as when various sets of constraints are active.

Dozens of papers have appeared in the literature describing various versions of MPC algorithms and applications since the report on Dynamic Matrix Control (DMC) by Shell [21], which started a resurgence of interest in this old idea. The activity is now more widespread than ever as summarized in a recent plenary by Gilbert [34]. An application-oriented survey is contained in [33], important recent contributions are due to Rawlings [64], Mayne [48, 47, 49], Polak [61, 62] and coworkers.

As part of MPC, a mathematical program is solved on-line in real-time. Because of the optimization, the performance for the nominal system (without uncertainty) as observed in simulations is usually excellent—as one would expect. The price one pays for this optimization approach is that it becomes exceedingly difficult to study stability, robust stability and robust performance with the modern analytical tools [50]. After many attempts by various researchers nominal stability has finally been addressed by Mayne [48, 47] in a general setting and very clean results for the case of a linear system with linear constraints are available from Rawlings [64].

Some interesting approaches have been proposed to address the robust stability and performance problems. The idea is to find—in a receding horizon manner—the manipulated variable moves which minimize the worst case tracking error predicted by a model in the family of possible plants. In addition system constraints are enforced for all models in the set [16, 49, 61, 62]. However, our simulation experience suggests that this objective often leads to a solution which is not useful from a practical point of view. Moreover, the resulting min-max problems are usually extremely complex numerically.

We had limited success [15, 16] assuming that the impulse response coefficients are affine functions of some uncertain parameters whose bounds define the set of LTI models. In this particular case the robust MPC problem can be formulated as a simple linear program. Uncertain gains in the elements of multi-input, multi-output systems can be handled in this form. Input (actuator positioning) uncertainty which has been found to be the dominant cause of poor performance for ill-conditioned systems, can be described as well in this manner.

In summary, notwithstanding a number of laudable attempts, we are far away from a set of analysis and synthesis tools which account for model uncertainty described in a manner which has proven useful in the linear context.

2.4.4 Antiwindup Bumpless-Transfer

The most common nonlinearity that control designers must deal with is control input saturation due to actuator limits. A saturation effectively turns-off the controller temporarily (no change in manipulated variable) and the controller states have to be adjusted properly ("Anti-Windup") as the manipulated variables come out of saturation. Without specific compensation for this, a poor transient response or even instability can result. Likewise a selector switches controllers in and out of a loop, in effect, switches them between "manual" and "automatic". A similar compensation is needed ("Bumpless-Transfer") to guarantee stability and a good response.

In standard Antiwindup Bumpless-Transfer (AWBT) schemes, actuator constraints are recognized by measuring the actual (constrained) implemented value of the manipulated variable or modeling the constraint. Other operating constraints are expressed in terms of standard control logic elements, in particular, selectors. The controller design proceeds in two steps where first an acceptable linear robust controller is designed neglecting all nonlinear loop elements and objectives. Then a compensation scheme is added which deals with the nonlinear issues.

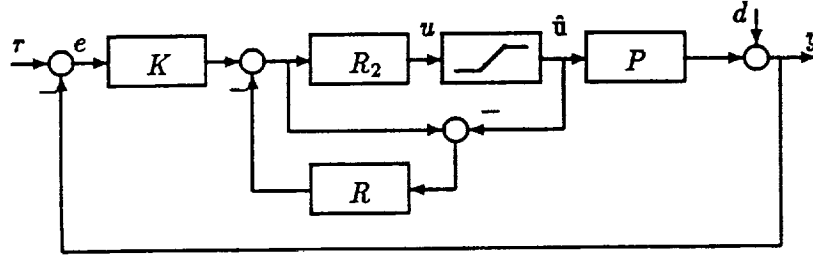


Figure 11: The feedback system with AWBT compensation, R , and directionality compensator, R_2

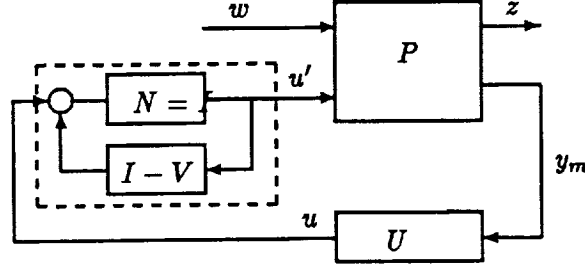


Figure 12: All linear AWBT schemes result from a coprime factorization of the controller $K = V^{-1}U$ with the two factors implemented as shown in the figure. Here N is a nonlinear (saturation) operator and V^{-1} results for the dotted block when $N = I$, i.e. there is no saturation.

Numerous different AWBT schemes have been proposed in the literature or have been implemented in commercial hardware and software. No general guiding principle is discernible; ad hoc engineering arguments are the rule. In [17, 18] we established a common framework for comparing the numerous different schemes. We found that *all* AWBT schemes presented in the literature can be put in the form shown in Figure 11, where K is the compensator designed for the linear system P , and R the AWBT compensator. R_2 is a “directionality- compensator” which is required only for MIMO systems which are sensitive to “input-uncertainty”, in particular ill-conditioned systems [27]. The various schemes differ in their choice of R .

In [14, 18] we postulated various properties of R and established in an axiomatic fashion a general class of AWBT compensators. We determined that all AWBT compensators satisfying the postulates can be presented in the form shown in Figure 12, where the controller $K(s)$ of the linear system has been factored into two coprime factors:

$$K(s) = V^{-1}(s)U(s)$$

To find the “best” AWBT compensator one has to search over all coprime controller factorizations. When looking for an appropriate objective function for the search we discovered some basic trade-offs which have to be addressed when designing AWBT compensators.

In particular, for an AWBT compensator with very good local performance, it may be impossible to establish global stability guarantees for the nonlinear system. For example, the internal model control implementation [50] of the controller is globally stable for any actuator nonlinearity. However, it is almost obvious that the performance of this AWBT compensator is not necessarily very good because the “controller” is entirely unaware if the manipulated variable has saturated or not. The compensator may also be very sensitive to noise in the measurement of the manipulated variable. It usually achieves good performance via high gain feedback from the measured/modeled

manipulated variables, which quickly resets the controller states as the manipulated variables move into and out of saturation. In turn, this high gain feedback can lead to the noise sensitivity.

Finally, the anti-windup schemes do not readily extend to multivariable systems. The classic concept of applying several single-loop controllers with standard AWBT compensators (as available on commercial computer control systems) to a multivariable system is not an acceptable answer either. Even if an unconstrained MIMO system can be controlled quite well with a set of single-loop controllers, the control system can fail in the presence of actuator constraints when the controllers are equipped with standard SISO AWBT compensators.

In our previous work we identified the following AWBT objectives:

1. Guarantee stability of the nonlinear system.
2. Optimize performance of the nonlinear system (in particular, minimize "directional sensitivity").
3. Optimize mode switching performance (minimize the "memory" of the controller).
4. Achieve linear performance recovery (avoid sensitivity to noise in the measurement of the manipulated variable).

We have quantitative analysis tools to measure to what extent these objectives are satisfied. We would like to develop a synthesis procedure which will generate an AWBT design which meets given performance requirements, stated in terms of the developed analysis methods, or establishes that no such AWBT design exists.

Performance as well as stability of the nonlinear system both with and without model error (1 & 2) can be assessed via an ∞ -norm test with appropriately optimized scaling matrices (μ -test [26]). The linear performance recovery (4) can be assessed directly with an ∞ -norm test as well. Minimization of memory (3) can be approached indirectly via minimization of an upper bound on the Hankel norm, again expressed through an ∞ -norm of the appropriate transfer matrix. In principle, an optimal trade-off between the various appropriately weighted norms can be achieved via Constrained Structure Control Synthesis [54] as suggested by [14]. This problem leads to a set of coupled Riccati equations for which effective solution techniques are not yet available. Furthermore, it is not clear that the various approximations introduced along the way preserved enough of the original problem characteristics to make the solution worthwhile.

2.4.5 Limitations of current theory

Current methods in nonlinear control rely on linear techniques for controller design. For some methods, such as output regulation, we explicitly use the linearization of the system about a single operating point to generate stable control laws. For feedback linearization based techniques, the system is transferred into a constant linear system over a (potentially large) region of the state space, where linear control methods can be applied.

The use of a *single* linear system for nonlinear control design is a serious handicap. Many nonlinear systems change behavior drastically at different operating points; forcing such a system to behave linearly by using nonlinear feedback requires large amounts of control effort. As a consequence, many nonlinear controllers are ill-conditioned and can generate large restoring signals even for small error signals. Furthermore, by transforming the inputs and states of the system, performance specifications become more complex. For example, a quadratic cost function on the original system may be transformed into a complicated nonlinear function. The use of quadratic

cost functions in the transformed variables can also be problematic: no physical intuition is available to help guide selection of weights in the feedback linearized coordinate system.

A common feature in almost all nonlinear controllers is the implicit or explicit generation of a state-space trajectory and subsequent stabilization of that trajectory. For all of the controller formulations given above, we can view the controller as a feedforward term which gives the nominal input required to move along the desired trajectory and an error correction term. The primary difference between the different methods is the form of the error correction term. For example, output regulation uses the linearization of the system about a single equilibrium point while feedback linearization uses a linear control law in an appropriate set of coordinates.

This two step approach to trajectory tracking can be carried one step further by completely decoupling the trajectory generation and asymptotic tracking problems. Given a desired output trajectory, we first construct a state space trajectory x_d and a nominal input u_d . The error system can then be written as a time-varying nonlinear system, depending on the nominal trajectory and input. Under the assumption that our tracking error remains small, we can linearize this time-varying system about $e = 0$ and stabilize the $e = 0$ state. One drawback to decoupling the trajectory generation and feedback portions of the controller is the need to find the nominal input and state trajectory *explicitly* before a feedback compensator can be generated. In the other approaches, this trajectory generation was implicit and hence a single controller could be used to track a large class of signals.

2.4.6 Nonlinear Inversion and Scheduling

To integrate nonlinear design with robust synthesis, we plan to use nonlinear inversion techniques to capture as many of the system nonlinearities as possible, and to use robust control techniques to account for the remaining nonlinearities and plant uncertainties. This approach has the advantage of using existing methods to develop working controllers, which can be tested on applications and used to further direct the theory, while at the same time providing a path to developing a formal mathematical structure for understanding robust control in a nonlinear context.

The first approach to generating a robust nonlinear controller will be to combine a standard feedback linearizing controller with a robust, linear, outer loop controller to provide robustness. Since most nonlinear systems are not exactly feedback linearizable, we will explore the effect of different approximations on the robust performance of the system. To analyze the robustness properties of a given control, we will simulate the closed loop control law on a class of trajectories and use existing μ techniques to evaluate the overall performance of the controller along a trajectory.

More complex strategies will rely on the use of LPV and LTV μ synthesis techniques to optimize the choices in the nonlinear inversion stage. In particular, we envision a methodology in which inversion is combined with gain-scheduling to create a controller which uses the geometric structure in different operating regions to the extent possible and minimizes the need for linear controllers to account for system nonlinearities. Nonlinearities that cannot be inverted out of the system will be accounted for using LPV and LTV synthesis methods. This will be accomplished by considering the remaining nonlinear system to be a linear system with the nonlinear dependency modeled as a parameter variation. This technique is conservative since the dynamics of the "parameters" will be ignored.

The eventual goal of this line of research is to push increasing amounts of the robustness analysis into the nonlinear controller and to develop a methodology in which robust performance in the presence of uncertainty can be analyzed in a nonlinear formalism.

3 Flight control and related application areas

We plan to develop several application areas to motivate and provide context for the theory. The largest of these applications will consist of a sequence experiments in flight control for rotary and fixed wing aircraft.

3.1 Pitch-axis Thrust-Vectoring

This section provides a brief review of current methods in the control of thrust-vectorred aircraft. Fundamental concepts can be found in the review article by Gal-Or [32].

3.1.1 Conventional control design for thrust-vectorred aircraft

Conventional control strategies for fixed-wing aircraft are primarily intended to augment the stability of the aircraft and decouple the body axes. The primary methodology is linear synthesis with the controller scheduled along an equilibrium manifold over the operating envelope. The scheduling parameters are generally chosen so that they are slowly varying and reflect large changes in the dynamic behavior of the system. Figure 13 shows a typical scheduled controller structure.

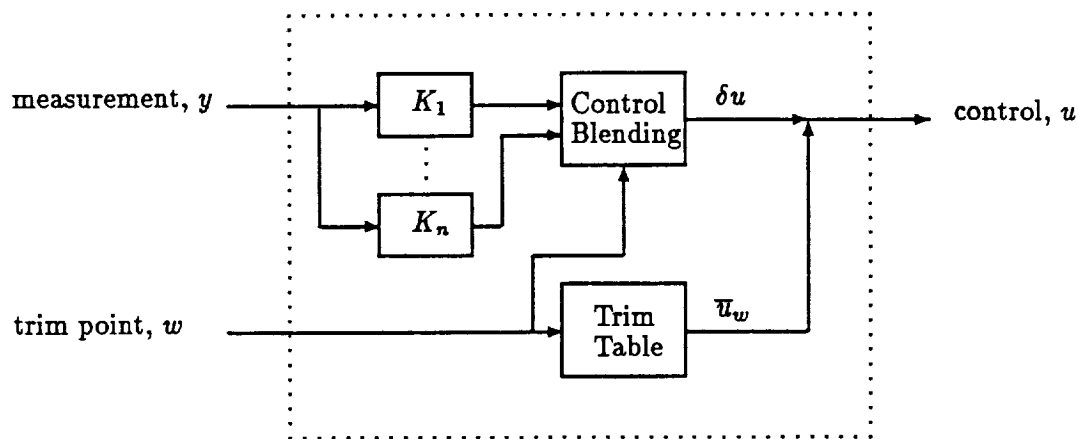


Figure 13: Typical Structure of a Gain-Scheduled Controller

The main problem with this control strategy is that in order to guarantee performance and stability for the global scheduled system the scheduling parameters need to be slowly varying and capture essential nonlinearities of the system [66, 68]. This generally precludes the use of large maneuvers over the operating envelope.

3.1.2 Supermaneuverability strategies for thrust-vectorred aircraft

When large maneuvers are required in the performance objective, new techniques must be considered. If exact global nonlinear models are available, these problems tend to be well-posed in the nonlinear framework.

As discussed previously, if the aircraft possesses certain geometric properties then coordinate transformations can be constructed which exactly linearize the input-output behavior. Once this coordinate transformation is made, a controller can be designed using standard linear theory. This technique has been shown to be quite successful solving local problems, such as output regulation.

Since the geometric conditions on the existence of the coordinate transformations are quite stringent this technique generally only yields local results even if global models are available. If the local region is large enough then problems such as trajectory tracking can be considered.

Motivated by this problem, approximate techniques have been developed so that the coordinate transformations approximately linearize the system over an equilibrium manifold [35, 41, 40]. These techniques provide schemes to linearize along equilibrium manifolds to order- ρ . That is, the system is linear near the equilibrium manifold up to polynomial terms of order ρ , where ρ is a design parameter. The main benefit of this technique is that the system can be linearized along an equilibrium manifold as opposed to near an equilibrium point. Hence steady-flight trajectory tracking can be done, as well as output regulation.

Since both of the above strategies rely on linearization near equilibrium manifolds they still provide no help when unsteady-flight maneuvers are desired. Recently techniques for *stabilizing* unsteady-flight maneuvers have been developed. They are generally used in systems where open-loop (pilot) control moves the aircraft through the unsteady-flight maneuver and the closed-loop control provides stability over this maneuver. This strategy relies heavily on bifurcation analysis [19]. Controllers are generally a composition of a linear controller, which extends the stable operating envelope, and a nonlinear controller, which stabilizes the periodic orbits resulting from bifurcations along the edge of the stable operating envelope [1]. The drawback to this technique is that the parameters generating the bifurcation are modeled as time-invariant; whereas, for a real system they will be time-varying. This can lead to complications in the analysis.

A major drawback of all of the preceding nonlinear methods is the requirement of having exact nonlinear models — e.g. there can be no modeling uncertainty. This means that one can generally only expect these control techniques to yield qualitative results; such as predicting the existence of specific bifurcations in the operating envelope or the general shape and size of the stable operating envelope. Quantitative results are much more dubious. Without uncertainty modeling and robustness the controllers can not be expected to perform quantitatively as the theory predicts.

The issue of robustness is the driving focus of our research program in nonlinear control. Our initial efforts will be directed at using available robust linear analysis and synthesis techniques to guide the free choices available in the existing nonlinear methods. The following experiment provides an excellent facility to explore this approach.

3.1.3 Description of the thrust-vectorred aircraft experiment

We plan to investigate the problem of robust control of nonlinear systems, with primary application in the area of supermaneuverability of high performance jet aircraft. The planned research is centered around a case study of the control of a thrust-vectorred aircraft, whose dynamics vary substantially over the operating regions of interest. When hovering, the aircraft is moving slowly and the dynamics are dominated by inertial forces and the complicated aerodynamic forces which depend heavily on the distance from the ground. In forward flight, the distance from the ground has little effect on the overall dynamics (excepting the dependence on air pressure variation), but Coriolis and centrifugal forces begin to play a role. Due to this strongly nonlinear behavior, linear-based design cannot be used to achieve high performance operation across all flight regimes.

The problem of nonlinear, robust control is a difficult one. It is unreasonable to expect that theoretical breakthroughs in this area will come easily. With this in mind, we have decided to focus on a specific problem in nonlinear robust control and use the results and intuition from that problem to guide future research in this area. The control of a thrust vectorred aircraft is intended to push nonlinear, robust design and to yield results with important practical applications.

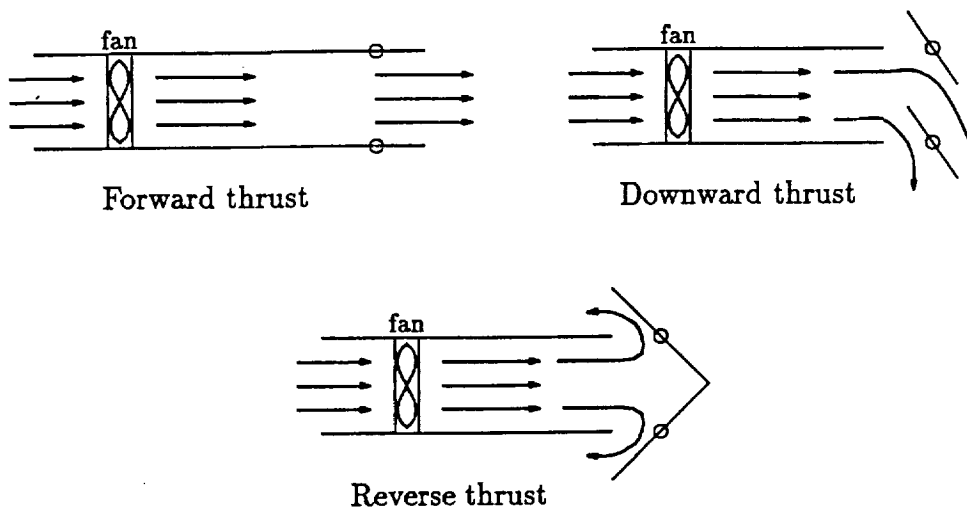


Figure 14: Operating modes of an experimental ducted fan engine

A particular area of interest is in trajectory tracking, especially for trajectories which do not lie in a single operating region. An example is the case where the ducted fan engine must transition between hover and forward flight. Since the dynamics in these two modes is very different, the use of nonlinear control is warranted. Furthermore, during the transition, the system is not operating near an equilibrium point and hence a classical linear system does not capture the system dynamics.

Since our system is one which is hard to model exactly, due in part to the use of thrust-vectoring as a control input, disturbances and model mismatch must be taken into account during the design process. Also, it is unlikely that a detailed description of the complete system dynamics will be available. Hence numerical identification of some system characteristics must be performed. Such identification procedures only give approximate descriptions of the plant, and the effect of these approximations must be taken into account in order to guarantee robust performance.

Because tools for robust linear control are well developed, we plan to search for controllers which analyze well in a linear context and simulate well nonlinearly. The process of building such controllers will help spotlight significant features of the problem. It will also serve as motivation for extending the software tools currently available to incorporate important aspects of new design methods which are developed. Examples might include provisions for generating parameterized controllers and directly converting the controller descriptions into executable code for a real-time control system.

An experimental setup is being built to study high performance pitch-axis control problems using a ducted-fan engine. The flaps on the engine can be used to generate forward and reverse thrust as well as up and down thrust. See Figure 14 for a schematic of the experimental engine in specific operating modes. The engine will be mounted on a 3-degree-of-freedom arm which allows horizontal and vertical translation as well as unrestricted pitch angle motion. Computer control of the system will be achieved using a DSP-based real-time operating system under development at Caltech. This system currently allows control algorithms written in the C programming language to be executed at software selectable rates.

This research is motivated primarily by the nonlinearities that result from large maneuvers; such as hover to forward flight transitions and post-stall recovery. Hence the control methodology employed in the experiments must be able to perform large maneuvers over non-equilibrium manifolds.

Such methodologies will need to incorporate nonlinear techniques to drive the large maneuvers and gain-scheduling to regulate and stabilize steady-flight trajectories; as well as addressing uncertainty so that the controller will be robust to modeling error.

3.2 Fixed Wing Free Flight

The goal of the work with pitch-axis thrust-vectoring is to move towards a full-scale remotely piloted vehicle (RPV) for studying large maneuvers during free flight. We plan to develop a complete setup for flight test and verification. The importance of such a setup lies in being able to understand more fully the types of nonlinearities that result from performing large maneuvers over a large operating envelope. Such maneuvers include post-stall recovery during high angle-of-attack, approaches at high angle-of-attack, and hover/forward flight transitions.

3.2.1 Applicability of RPV research to full-scale vehicles

Development of flight control technologies using sub-scale RPV systems has many benefits. The critical elements of model uncertainty and disturbances are included in the development process. There is low relative cost with respect to traditional flight test and verification programs. The flexible system allows multiple technology development programs. Using sub-scale vehicles requires consideration of the applicability of results to full-scale vehicles. The vehicle must be large enough to minimize scaling effects (e.g. Reynolds number), provide manned vehicle credibility, assure adequate power plant for thrust-vectoring, and accommodate necessary subsystems and instrumentation. It must also be small enough to minimize vehicle cost, assure ease of launch/recovery, and minimize reconfiguration turnaround time. Figure 15 compares key vehicle and system requirements for typical sub-scale and full-scale vehicles.

	FULL SCALE	SUB-SCALE	
		REQUIRED	ACHIEVABLE
Computer Frame Rate	100 Hz	200 Hz	> 1000 Hz
Actuator Bandwidths	40 – 50 rad/sec	100 rad/sec	100 – 300 rad/sec
Actuator Rates	60 – 100 deg/sec	120 deg/sec	120 – 360 deg/sec

Figure 15: Vehicle and System Requirements

Of critical importance in this program are vehicle agility limitations as a result of improved stability and robustness, and control effectiveness during engine stall. This requires novel and easily reconfigurable control architectures to be considered. The adaptability of small, reliable engines for thrust-vectoring, the use of thrust-vectoring for increasing the stable operating envelope, and the successful understanding and exploitation of unique aerodynamic phenomena are key issues. Many non-traditional control alternatives will also be considered, such as leading edge vortex manipulation, dynamic lift exploitation, functional control redundancy with aerodynamic surfaces, jettisonable wing extensions (agile cruise missile), and the application of micromachines.

The dominating performance issues in this research program include: power compensation during approach for a highly backside, unstable aircraft, reconfigurable controls, ride quality improvements, and stabilization and performance for large maneuvers over large operating envelopes. This type of highly acrobatic flight exploits technical advances in:

- Digital flight control systems software/hardware/control laws

- Real-time telemetry and remote control
- High bandwidth actuators
- Compact flight control computers
- Temperature tolerant carbon materials
- Advanced composite materials, structures, and manufacturing
- Miniature sensors and instrumentation
- Computational design methodologies
- Unsteady flow test facilities and techniques

3.2.2 Description of the free flight experiment

We hope to develop an experimental system through collaboration with NASA and the Aerospace Industry. We will focus on a statically unstable, high angle-of-attack vehicle. The candidate sub-scale RPVs include the NASA F-18, NASA X-31, and a modular generic attack/fighter aircraft. These vehicles will include a "drop-in" flight computer, sensor, and battery package. Typical vehicle and system characteristics are pitch-axis instabilities in the range of 0.3–1 second time to double amplitude for the sub-scale vehicle. Refer to Figure 16 for the typical RPV control system structure.

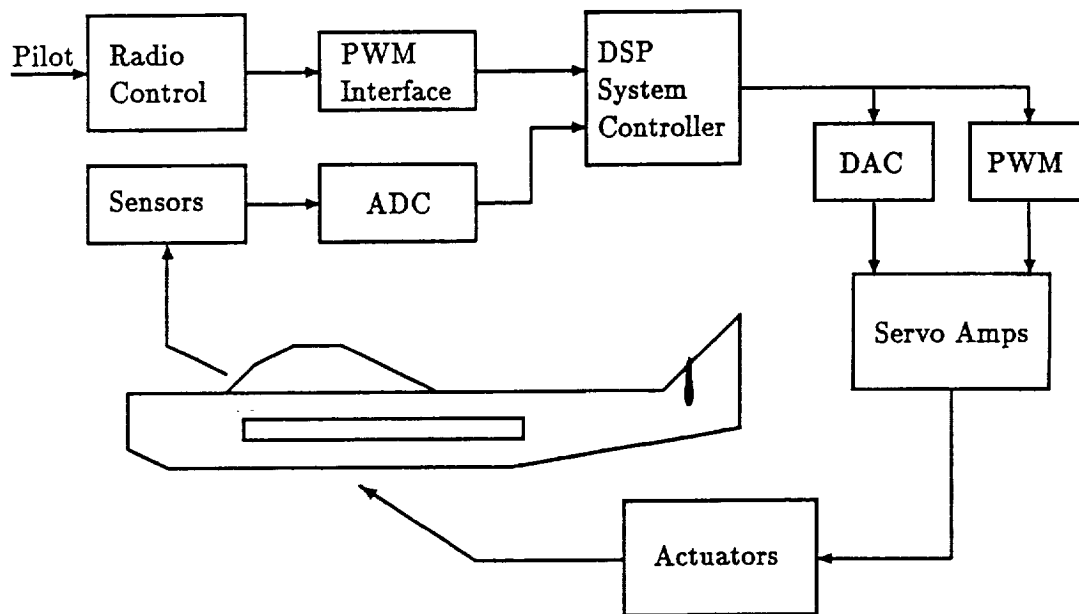


Figure 16: Typical RPV Control System Structure

Research on RPV vehicles will lead to a proof of principle for agile, high speed air vehicles: improved combat supercruise range, reduced vehicle signatures multiply force effectiveness, and superior low speed/transitional maneuvering capability. This will provide a programmatic model for future development and flight test efforts.

3.3 Guidance, Navigation, and Control of Helicopters

3.3.1 Description of the experimental setup

This section provides a description of the goals of this research program with respect to autonomous guidance, navigation, and control of helicopters. We plan to follow the experimental setup developed by George Mason University (GMU) for Steve Suddarth and the AFOSR FAME program. The FAME experimental setup consists of an electric helicopter attached to a 3 degree-of-freedom gimbaled "manipulator" at the end of a 3 degree-of-freedom arm. The arm is counter-balanced and has fairly low inertia. The purpose of the arm is to provide a mechanical constraint on the motion of the helicopter and facilitate measurement of the position and attitude of the helicopter. The reachable volume of this arm is roughly a hemisphere with a six foot radius. The measurements of the 6 degrees-of-freedom is via potentiometers. These measurements along with the forward kinematics of the arm can be used to compute the position and attitude of the helicopter. Each degree-of-freedom has a mechanical stop preventing full rotation. The interface to the helicopter actuators and potentiometer measurements is through a microprocessor-based circuit board with a serial interface. The host computer can request the potentiometer measurements and send actuator commands to the helicopter through this serial interface. All actuator computations are performed on the host computer. We outline several substantial problems with this setup and our proposed solutions.

As mentioned above, the host computer interfaces with the microprocessor-based circuit board through a serial interface. Using the FAME software version 2.1 we were bottle-necking the circuit board at closed-loop bandwidths of around 5 Hz without any computation on the host. Output computation would further reduce this bandwidth. Sampling period drift is quite difficult when using a serial interface, and changing the input/output configuration of the system by augmenting actuators/sensors is not straightforward. We solve this problem by using a general digital signal processing (DSP) system with a rich input/output (I/O) interface, including analog-to-digital (ADC) and digital-to-analog (DAC) conversion, pulse-width modulation (PWM) inputs and outputs, quadrature input decoding, and digital I/O. With this system we can interface to the helicopter actuators and sensors directly and easily achieve true real-time closed-loop bandwidths in excess of 100 KHz (which is a few orders of magnitude faster than necessary for flight control). In addition, it is very easy to modify the system for any I/O configuration which uses analog, digital, or PWM signals or quadrature inputs. As an example we can easily incorporate a pilot in the closed-loop system by feeding the PWM servo commands the pilot generates with an RC transmitter into our DSP system and processing them as standard PWM inputs. Sensor augmentation for measuring rates, accelerations, etc. is easily accomplished by using ADCs. Generating the controller code and selecting sampling rates is easily accomplished by either modifying the C source code (for nonlinear systems) or using the standard linear system interface through Matlab data files.

Since the FAME system uses potentiometers for measurements the full motion of the arm is lost since no joint can freely rotate (due to mechanical stops). This means, for example, that the helicopter can not fly circles or yaw continuously. This is a serious limitation when studying trajectory tracking problems and limits this system to primarily studying control problems near hover. We solve this problem by using slip rings and optical shaft encoders at all fully rotating joints. This allows wires to be routed through a rotating joint and measurement of the joint angles without discontinuities or dead band. The optical shaft encoders are easily processed through the quadrature encoder inputs in the DSP system.

We feel that this experimental setup provides an ideal starting point for studying the problems

associated with autonomous guidance, navigation, and control of helicopters in a research laboratory. Experiments outside the laboratory in free-flight can be carried out with the same helicopter and DSP system with the addition of a non-contact position and attitude measurement system. One such system is included in the budget. The experiments we plan to perform are outlined in the following sections.

3.3.2 Experiments near hover

The purpose of studying control of the helicopter near hover is to provide a stable autopilot platform for performing larger maneuvers. To this end we will study a progression of experiments leading to an autopilot which can robustly regulate a hover position and precisely track yaw attitude and vertical position. The progression of experiments is:

- LPV identification near hover (quasi steady-state models)
- Regulation of hover
- Yaw attitude tracking
- Vertical position tracking
- Repetition of experiments with disturbances and modeling error

Initially experiments will focus on identifying linear parameter-varying (LPV) models near hover. These models will be used along with state-space nonlinear models obtained via first principles [42, 43, 45, 67, 74]. Then controllers are designed through iterations involving simulation and flight test in order to achieve good performance and robustness.

3.3.3 Experiments with large maneuvers

The "training arm" system can be used to study control problems involving large maneuvers as well. In particular, trajectories lying within the 6 foot hemispherical operating volume of the arm are easily studied. Larger trajectories can be studied in an *ad hoc* fashion by attaching the arm to a moving platform and designing control systems which "track" the moving platform. In this way the operating volume of the arm is greatly increased, opening up a much larger realm of experimental problems which can be studied. The progression of experiments is:

- LPV identification along *a priori* large maneuver trajectories
- Trajectory tracking
- Repetition of experiments with disturbances and modeling error

As with the experiments near hover, initially experiments will focus on identifying LPV models along the large maneuver trajectories. Additional knowledge about the system dynamics can be included as nonlinear perturbations on the LPV model. Controllers will be designed for these systems so that they have robust performance when linearized and simulate well nonlinearly. Choice of the trajectory is the key design issue. The amount of nonlinearity in the model will vary dramatically depending on the types of trajectories chosen. This will enable a progression of increasingly harder control problems, the results of which will drive the important theoretical issues.

3.3.4 Autonomous guidance and navigation during free-flight

All of the experiments discussed thus far have been mechanical restricted to a relatively small operating volume since the "training arm" is being used. This arm not only restricts the operating volume but also changes the dynamics and adds mechanical damping. It is, therefore, of interest to replicate the experiments performed on the arm in a free-flight setting. The experimental setup for free-flight involves the same helicopter and DSP system; but the measurement sensors will change. The addition of off-board position and attitude measurements and rate gyros becomes a necessity. There are commercially available systems which are small and light enough to be used. These systems are included in the budget.

The autonomy of the control system is largely dependent in knowledge of the external environment. Either *a priori* knowledge or measurements must be available. Measurement of the external environment is generally obtained through both active and passive imaging in the ultrasonic, infrared, and visual spectrum. Development of such a system is a long-term goal; but, initially all information about the external environment that is needed for autonomy will be made available to the control system.

The chief advantage in studying problems in true free-flight lies in the freedom of trajectories that can be obtained and the ability to select highly nonlinear maneuvers. Maneuvers which lie far away from equilibrium points of the helicopter, such as coordinated turns and loops (which are possible with RC helicopters) are of particular interest.

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